

## The Untitled Binary Sequence

I was considering the Hilbert curve and its extension to higher dimensions. There seems to be some discrepancy about how to orient the individual boxes in more than two dimensions. The problem arises from the following issue: By making $90^{\circ}$ (orthogonal) rotations, a square can only be oriented 4 different ways on a plane, but a cube can be oriented 24 ways in 3 -space, and a tesseract can be oriented 196 ways in 4 -space.

Although there are likely very simple rules that can be established that are consistent across all levels, I left that problem alone and instead focused on the properties of the basic element of a space filling Hilbert curve. In two dimensions it is like a horseshoe, and in three dimensions it is a folded and extended version of that horse-shoe.


This shape is the path used to pass through every vertex in an n -cube. If one tries to trace the edges of a six-sided die with a marker, for example, they will find the same shape as in the image on the right, although the start and end points may be different if it is not a closed loop.

Because I am dealing with the simplest element, I can assign every dimension to a bool/bit: Left or Right, Up or Down, Forward or Back, Out or In. I can then address any location-vertex, as it is - a binary value. In my 3-cube above, the bottom right corner the line starts at can be 000. It travels back to 001, then up to 011, then forward to 010, left to 110 , back to 111, down to 101, and finally forward to 100.

I found this results in a new counting sequence for binary. It follows the rule that only one place value (or dimension) can be changed at a time. The process is systematic. I thought about calling this the orthogonal binary sequence, but that already has a meaning in electronics. The applications of this sequence are unknown to me today.

These numbers can be converted to the original binary with this algorithm:

Os are read as 0 s and 1 s are read as 1 s . After reading a 1 , the opposite becomes true, until after another 1 is read.

| SETTING | READ | OUTPUT | NEW SETTING |
| :---: | :---: | :---: | :---: |
| $0=0,1=1$ | 0 | 0 | $0=0,1=1$ |
| $0=0,1=1$ | 1 | 1 | $0=1,1=0$ |
| $0=1,1=0$ | 0 | 1 | $0=1,1=0$ |
| $0=1,1=0$ | 1 | 0 | $0=0,1=1$ |

The original binary can be converted to this new form the same way, but with the added rule that every time the values of the 1 s and 0 s are swapped, the digit that triggers that swap is changed. I.e. The reversal occurs every time the digit read is different from the last.

I've looked at some of the patterns emerging from the sequence. The map in the corner, read from right to left and top to bottom, indicates the replacing digits ( $0 s$ and 1s) with black and white pixels. The values on the far-left column of this page indicate the place value that is toggled (log2). Keeping track of these values is a highly effective way to keep track of the counting sequence itself. It resembles the fractional inch marks on a ruler, but rather than being fractal it grows outward to infinity. This also exists in the normal binary.

